Prof. M. Magill **Econ 603 - Problems 10** Spring 2005

given: Wed March 9; due: Monday April 13

Subject: Spot Financial Market Equilibrium and Security Pricing

1. Let's go back to a model of a *real economy.* Now we know why we can leave out monetary considerations from the model: we leave them out because the model is much simpler, and we know that the resulting allocation would be the same, even if we did introduce the monetary equations. When economists talk about real economies you often hear them say: lets normalize by taking prices to lie in the simplex (for each state of nature). Well, thats just a way of saying that price levels (and hence money) dont matter in a real economy. In particular, consider a (real) exchange economy with two periods ($t = 0, 1$) and uncertainty described by S states of nature at date 1. There are L goods at each date and agent i has the initial endowment $\omega^i \in \mathbb{R}_+^{L(S+1)}$. The preferences of each agent are characterized by a utility function $u^i : \mathbb{R}_+^{L(S+1)} \longrightarrow \mathbb{R}$ which is smooth, strictly quasi-concave and has indifference surfaces which do not intersect the axes.

(a) Define a spot-financial market equilibrium for the economy, where $p = (p_0, \ldots, p_S)$ is the vector of spot prices, $q = (q_1, \ldots, q_J)$ the vector of security prices and V is the $S \times J$ matrix of returns on the securities. Notice how you dont have any monetary equations.

(b) Suppose the financial markets are complete. Using the agents' first order conditions at an equilibrium and the first order conditions for Pareto optimality, show that the equilibrium is Pareto optimal.

(c) Suppose as in (b) that the financial markets are complete, that the first security is the riskless bond ($V^1 = (1, \ldots, 1)$) and that agents have VNM utility functions

$$u^i(x^i) = v_0^i(x_0^i) + \sum_{s=1}^{S} \rho_s v^i(x_s^i)$$

where $\rho_s > 0$ is the probability of state s ($s = 1, \ldots, S$) and $v^{i\prime} > 0, v^{i\prime\prime} < 0$. Show that the equilibrium price of security j can be written as

$$q_j = \frac{E(V^j)}{1+r} + \text{ cov } (V^j, \frac{\pi}{\pi})$$

where r is the equilibrium interest rate.

(d) Let $w_1 = (w_1, \ldots, w_S)$ denote random aggregate output, where $w_s = \sum_{i=1}^{I} \omega_s^i$ denotes aggregate output in state s. Show that there exists a strictly decreasing function g such that the equilibrium price can be written as

$$q_j = \frac{E(V^j)}{1+r} + \text{ cov } (V^j, g(w_1))$$

How do we define the *risk premium* on security j. Explain the economics in terms of the nature of the stochastic dependence between w_1 and V^j. For what kind of securities do you expect the sign of the premium to be positive and why?

(e) Suppose the economy is a *bond-equity economy* i.e. there is a riskless security (call it security 0 for convenience) and the remaining securities are equity contracts for K firms (whose production plans y^k are taken as exogenously fixed for $k = 1, \ldots, K$). Agents only have endowments at date 0 ($\omega_0^i > 0, \omega_s^i = 0, s = 1, \ldots, S$). Thus aggregate output in state s (at date 1) is $w_s = \sum_{k=1}^{K} y_s^k, s = 1, \ldots, S$. Define the rates of return on the "market porfolio" and on equity k by

$$R_m = \frac{w_1}{\sum_{k=1}^{K} q_k}, \quad R_k = \frac{V^k}{q_k}, \quad k = 1, \ldots, K$$

Suppose agents have quadratic (date 1) utility functions

$$v^i(x_s^i) = -\frac{1}{2}(x_s^i - \alpha_i)^2$$

with $\alpha_i >$ max $\{w_0, \ldots, w_S\}$. Show that

$$E(R_k) - R = \beta_k(E(R_m) - R)$$

where $R = 1 + r, \beta_k = \frac{\text{cov } (R_k, R_m)}{\text{var } R_m}$. Give an economic interpretation for this risk pricing formula. How do you interpret the coefficient β_k? What would you expect β_k to be (i) for the Southern California Gas Company and (ii) for the Ford Motor Company? Why the difference?

Note that the CAPM formula which you have just derived tells us what the risk premium on security j is, given its volatility (β_k), and given the quantity $E(R_m) - R$, which is called the *equity premium* on the security market as a whole. But CAPM tells us nothing about the magnitude of the equity premium on the market as a whole: this has been the subject of a lot of controversy. Have you heard about it? What is the so-called *equity premium puzzle* and why do some economists get so concerned about it?

2. Consider the following two-person general equilibrium economy. There are two dates ($t = 0, 1$), two states of nature ($s = 1, 2$) at date 1 and two goods, leisure and consumption. Decisions are made at date 0 and for simplicity there is no leisure or consumption at date 0, only at date 1. For $i = 1, 2$, let

$$x^i = (x_1^i, x_2^i) = (x_{L1}^i, x_{C1}^i, x_{L2}^i, x_{C2}^i)$$

denote agent i's leisure and consumption choice in states 1 and 2. Agent 1 has the utility function

$$U^1(x^1) = \rho_1 x_{C1}^1 + \rho_2 x_{C2}^1$$

where $\rho_s > 0$ is the probability of state s, and the initial endowment $\omega^1 = (0, \bar{\omega}, 0, \bar{\omega})$ where $\bar{\omega}$ is a large positive number. Agent 2 has the utility function

$$U^2(x^2) = \rho_1(x_{L1}^2 + \ln(x_{C1}^2)) + \rho_2(x_{L2}^2 + \ln(x_{C2}^2))$$

and the initial endowment $\omega^2 = (24, 0\ ,\ 24, 0)$. Agent 2 is also the sole owner of a firm with a state dependent production set given by

$$Y = \left\{ y \in \mathbb{R}^4 \mid y = (-1, \bar{c}, -1, \underline{c})L,\ L \geq 0 \right\}$$

where $0 < \underline{c} < \bar{c}$.

(a) Define an equilibrium with complete contingent markets (CM) for this economy.

(b) Find the CM equilibrium: what interesting properties does it have with respect to risk sharing and the equilibrium prices?

(c) Define an equilibrium in which the only markets are spot markets for the goods in each state at date 1: call it an SM equilibrium. Note that since the labor input does not depend on the state, the labor input decision must be made at date 0 before the state of nature is revealed. Find the SM equilibrium.

(d) Compare the CM and SM equilibria: in which are the agents better off and why?